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ON THE TIDES OF THE ARCTIC SEAS: PART IX.—ON THE TIDES OF THE NORTH-WESTERN ENTRANCE OF ROBESON CHANNEL, GRINNELL LAND. BY REV. SAMUEL HAUGHTON, M.D., Fellow of Trinity College, Dublin.

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## XV.

# ON THE TIDES OF THE ARCTIC SEAS: Part IX.—ON THE TIDES OF THE NORTH-WESTERN ENTRANCE OF ROBESON CHANNEL, GRINNELL LAND.

By REV. SAMUEL HAUGHTON, M.D., Fellow of Trinity College, Dublin.

[Read November 12, 1894.]

The Tidal Observations discussed in the present Paper were made on board H. M. S. "Alert," from September 29, 1875, to October 23, 1875, and from December 12, 1875, to January 11, 1876.

The ship was moored in Lat. 82° 27′ N., Long. 61° 22′ W., and rose and fell with the ice-floe.

The observations were intended to have been made at each hour of the day, and to have been discussed by me in accordance with Fourier's Theorem. Owing to the imperfect nature of the observations, it has been found impossible to draw any conclusions respecting the mean tide level, or the Diurnal Tide, whether solar or lunar.

The Semidiurnal Tide is represented by the formulæ

$$\begin{aligned} 12A_2 = {} & (F_0 + F_{12}) - (F_6 + F_{18}) \\ & + \left\{ \begin{matrix} (F_1 + F_{23}) + (F_{11} + F_{13}) \\ -(F_5 + F_{19}) - (F_7 + F_{17}) \end{matrix} \right\} \cos 2\phi \\ & + \left\{ \begin{matrix} (F_2 + F_{22}) + (F_{10} + F_{14}) \\ -(F_4 + F_{20}) - (F_8 + F_{16}) \end{matrix} \right\} \cos 4\phi. \end{aligned}$$

$$\begin{aligned} 12B_2 = {} & (F_3 + F_{15}) - (F_9 + F_{21}) \\ & + \left\{ \begin{matrix} (F_1 + F_5) + (F_{13} + F_{17}) \\ -(F_7 + F_{11}) - (F_{19} + F_{23}) \end{matrix} \right\} \sin 2\phi \\ & + \left\{ \begin{matrix} (F_2 + F_4) + (F_{14} + F_{16}) \\ -(F_8 + F_{10}) - (F_{20} + F_{22}) \end{matrix} \right\} \sin 4\phi, \end{aligned}$$

where $$\phi = \frac{360}{24} = 15°.$$

The following Table has been calculated from the observed heights by means of the foregoing and similar equations, familiar to those who use Fourier's Functions.

| Day. | $A_0$. | $A_1$. | $B_1$. | $A_2$. | $B_2$. | $A_3$. | $B_3$. |
|---|---|---|---|---|---|---|---|
| 1875. September. | *Inches.* | *Inches.* | *Inches.* | *Inches.* | *Inches.* | *Inches.* | *Inches.* |
| 29 | 37·2 | − 0·8 | + 1·9 | + 11·5 | − 7·1 | + 0·6 | − 0·5 |
| 30 | 35·7 | − 0·6 | + 0·8 | + 13·8 | − 5·8 | − 0·2 | + 0·5 |
| October. | | | | | | | |
| 1 | 36·8 | − 1·7 | + 1·2 | + 13·6 | − 2·7 | + 1·0 | + 0·2 |
| 2 | 34·1 | − 0·7 | + 3·0 | + 13·3 | + 1·6 | + 0·8 | 0·0 |
| 3 | 33·2 | − 1·0 | − 0·6 | + 10·6 | + 3·3 | + 0·2 | + 0·7 |
| 4 | 36·1 | − 2·1 | − 0·7 | + 9·0 | + 4·8 | + 0·4 | + 0·3 |
| 5 | 39·3 | − 1·9 | − 0·7 | + 6·4 | + 5·6 | − 0·1 | 0·0 |
| 6 | Heavy snow drift. | | | | | | |
| 7 | Heavy snow drift. | | | | | | |
| 8 | | | | | | | |
| 9 | | | | | | | |
| 10 | 34·6 | − 0·4 | 0·0 | − 2·7 | − 3·8 | − 0·2 | − 0·6 |
| 11 | | | | | | | |
| 12 | | | | | | | |
| 13 | | | | | | | |
| 14 | 26·9 | − 5·0 | − 2·1 | + 8·2 | − 6·2 | + 2·5 | + 0·5 |
| 15 | 21·2 | − 2·7 | + 0·1 | + 14·1 | − 8·3 | + 0·2 | − 0·8 |
| 16 | 24·8 | − 2·5 | + 1·4 | + 16·0 | − 3·2 | − 0·3 | + 0·9 |
| 17 | 24·8 | − 3·2 | + 1·6 | + 15·1 | + 1·3 | + 0·5 | + 0·4 |
| 18 | 25·3 | − 2·7 | + 1·2 | + 12·3 | + 5·4 | + 0·3 | + 0·8 |
| 19 | 25·5 | − 2·8 | + 1·2 | + 8·0 | + 6·9 | − 0·2 | + 1·5 |
| 20 | 27·9 | − 2·3 | − 0·8 | + 3·0 | + 5·8 | − 0·7 | + 0·2 |
| 21 | 28·3 | − 2·2 | + 0·7 | − 0·6 | + 5·5 | − 0·1 | + 0·9 |
| 22 | 24·7 | − 0·9 | − 0·1 | + 4·1 | + 1·8 | − 0·1 | + 0·2 |
| 23 | 20·4 | − 1·0 | + 1·0 | − 3·8 | − 3·3 | − 0·2 | + 0·7 |
| 24 | Tidal observations ceased on October 24, and were not resumed until December 11. | | | | | | |
| December. | | | | | | | |
| 12 | 43·6 | − 2·0 | + 6·4 | + 9·9 | − 10·6 | + 1·1 | − 0·5 |
| 13 | 41·6 | − 2·9 | + 3·1 | + 13·9 | − 6·8 | + 1·0 | − 0·1 |
| 14 | 42·9 | − 3·9 | + 3·4 | + 15·3 | ± 0·0 | + 1·1 | + 0·5 |

| DAY. | $A_0$. | $A_1$. | $B_1$. | $A_2$. | $B_2$. | $A_3$. | $B_3$. |
|---|---|---|---|---|---|---|---|
| 1875. DECEMBER. | *Inches.* | *Inches.* | *Inches.* | *Inches.* | *Inches.* | *Inches.* | *Inches.* |
| 15 | 45·3 | − 3·6 | + 0·4 | + 13·0 | + 4·9 | + 0·9 | + 0·8 |
| 16 | 45·8 | − 2·9 | + 2·1 | + 9·3 | + 10·0 | + 0·6 | + 1·5 |
| 17 | 43·0 | − 1·7 | + 1·4 | + 3·7 | + 11·3 | + 0·2 | + 0·7 |
| 18 | 42·6 | − 1·6 | + 1·1 | − 1·3 | + 10·3 | + 0·1 | + 0·6 |
| 19 | 40·5 | 0·0 | + 0·9 | − 4·8 | + 6·4 | + 0·2 | + 0·7 |
| 20 | 39·3 | + 0·7 | + 1·8 | − 7·0 | + 1·8 | − 0·2 | ± 0·0 |
| 21 | 41·9 | − 0·3 | + 1·7 | − 6·4 | − 2·3 | − 0·4 | − 0·4 |
| 22 | 41·5 | + 0·7 | + 3·7 | − 4·8 | − 5·5 | − 0·3 | − 0·4 |
| 23 | 42·1 | + 0·2 | + 1·9 | − 2·8 | − 8·5 | − 0·5 | − 0·7 |
| 24 | 41·0 | − 0·5 | + 6·6 | + 1·9 | − 8·3 | + 0·2 | + 0·2 |
| 25 | 39·0 | − 1·0 | + 2·7 | + 4·4 | − 9·5 | − 0·5 | − 0·6 |
| 26 | 43·0 | − 1·5 | + 3·5 | + 6·9 | − 7·3 | + 0·1 | − 0·4 |
| 27 | 45·3 | − 0·4 | + 5·4 | + 7·9 | − 4·3 | − 1·6 | − 0·3 |
| 28 | 46·6 | − 3·2 | + 1·7 | + 11·3 | − 3·7 | + 0·4 | − 0·8 |
| 29 | 47·0 | − 3·0 | + 3·2 | + 12·5 | + 0·8 | + 0·6 | + 0·1 |
| 30 | 43·7 | − 2·7 | + 3·2 | + 11·6 | + 4·3 | + 0·9 | + 1·3 |
| | Tidal observations ceased on December 30, and were resumed on January 8. | | | | | | |
| 1876. JANUARY. | | | | | | | |
| 8 | 30·3 | + 1·1 | + 4·4 | − 1·9 | − 10·2 | + 1·2 | − 1·9 |
| 9 | 37·4 | − 3·3 | + 2·8 | + 2·8 | − 11·8 | + 0·1 | − 1·9 |
| 10 | 27·9 | − 3·7 | + 7·8 | + 10·1 | − 9·6 | + 2·1 | + 0·3 |
| 11 | 23·5 | − 2·2 | + 1·1 | + 13·0 | − 5·4 | + 1·2 | − 0·5 |
| | Tidal observations ceased altogether. | | | | | | |

*Semidiurnal Tides.*

The coefficients $A_2$, $B_2$ belong to the Semidiurnal Tide, and

$$A_2 = S \cos 2i_s \cos^2 \bar{\sigma} \qquad (1)$$
$$+ M\left(\frac{p}{p_m}\right)^3 \cos^2 \bar{\mu} \cos 2(m - s - i_m),$$

$$B_2 = S \sin 2i_s \cos^2 \bar{\sigma} \qquad (2)$$
$$- M\left(\frac{p}{p_m}\right)^3 \cos^2 \bar{\mu} \sin 2(m - s - i_m).$$

If we had hourly observations made for 31 days in succession, by adding the values of $A_2$ and $B_2$, the Lunar Tide would disappear, and we should have the Solar Tide isolated, and easy to discuss.

In the absence of 31 days consecutive hourly observation, we must use other methods less perfect than those of Fourier.

From equations (1) and (2) we find

$$-\tan 2(m-s-i_m) = \frac{B_2 - S\sin 2i_s \cos^2\bar{\sigma}}{A_2 - S\cos 2i_s \cos^2\bar{\sigma}} \qquad (3)\ldots.\text{Times.}$$

and

$$(A_2 - S\cos 2i_s \cos^2\bar{\sigma})^2 + (B_2 - S\sin 2i_s \cos^2\bar{\sigma})^2 = M^2\left(\frac{p}{p_m}\right)^6 \cos^4\bar{\mu}. \qquad (4)\ldots.\text{Heights.}$$

The first of these equations becomes, on the days on which $\tan 2(m-s) = 0$, simply

$$\tan 2i_m = \frac{B_2 - S\sin 2i_s \cos^2\bar{\sigma}}{A_2 - S\cos 2i_s \cos^2\bar{\sigma}}. \qquad (5)$$

or,

$$\tan 2i_m = \frac{B_2 - y\cos^2\bar{\sigma}}{A_2 - x\cos^2\bar{\sigma}}, \qquad (6)$$

writing

$$x = S\cos 2i_s,$$
$$y = S\sin 2i_s.$$

Fortunately, of the days on which hourly observations were made, there were seven, on which equations (6) hold good.

*September* 29, 1875.

$$m \sim s = 0^h\,0^m, \qquad \cos^2\sigma = 0{\cdot}998,$$
$$A_2 = +11{\cdot}5, \qquad B_2 = -7{\cdot}1,$$
$$\tan 2i_m = \frac{0{\cdot}998\,y + 7{\cdot}1}{0{\cdot}998\,x - 11{\cdot}5}. \qquad (a)$$

*October* 14, 1875.

$$m \sim s = 11^h\ 45^m, \qquad \cos^2\sigma = 0{\cdot}987,$$

$$A_2 = +\,8{\cdot}2, \qquad B_2 = -\,6{\cdot}2,$$

$$\tan 2i_m = \frac{0{\cdot}987\,y + 6{\cdot}2}{0{\cdot}987\,x - 8{\cdot}2}. \qquad (b)$$

*October* 20, 1875.

$$m \sim s = 17^h\ 45^m, \qquad \cos^2\sigma = 0{\cdot}968,$$

$$A_2 = +\,3{\cdot}0, \qquad B_2 = +\,5{\cdot}8,$$

$$\tan 2i_m = \frac{0{\cdot}968\,y - 5{\cdot}8}{0{\cdot}968\,x - 3{\cdot}0}. \qquad (c)$$

*December* 12, 1875.

$$m \sim s = 12^h\ 4^m, \qquad \cos^2\sigma = 0{\cdot}846,$$

$$A_2 = +\,9{\cdot}9, \qquad B_2 = -\,10{\cdot}6,$$

$$\tan 2i_m = \frac{0{\cdot}846\,y + 10{\cdot}6}{0{\cdot}846\,x - 9{\cdot}9}. \qquad (d)$$

*December* 18, 1875.

$$m \sim s = 17^h\ 45^m, \qquad \cos^2\sigma = 0{\cdot}843,$$

$$A_2 = -\,1{\cdot}3, \qquad B_2 = +\,10{\cdot}3,$$

$$\tan 2i_m = \frac{0{\cdot}843\,y - 10{\cdot}3}{0{\cdot}843\,x + 1{\cdot}3}. \qquad (e)$$

*December* 27, 1875.

$$m \sim s = 0^h\ 0^m, \qquad \cos^2\sigma = 0{\cdot}843,$$

$$A_2 = +\,7{\cdot}9, \qquad B_2 = -\,4{\cdot}3,$$

$$\tan 2i_m = \frac{0{\cdot}843\,y + 4{\cdot}3}{0{\cdot}843\,x - 7{\cdot}9}. \qquad (f)$$

*January* 10, 1876.

$$m \sim s = 11^h\ 54^m, \qquad \cos^2\sigma = 0{\cdot}859,$$

$$A_2 = +10{\cdot}1, \qquad B_2 = -9{\cdot}6,$$

$$\tan 2i_m = \frac{0{\cdot}859\,y + 9{\cdot}6}{0{\cdot}859\,x - 10{\cdot}1}. \qquad (g)$$

If we reduce these seven equations to their simplest form, we find

$$y - x\tan i_m = -11{\cdot}50\tan i_m - 7{\cdot}11. \qquad (a)$$
$$y - x\tan i_m = -8{\cdot}30\tan i_m - 6{\cdot}28. \qquad (b)$$
$$y - x\tan i_m = -3{\cdot}09\tan i_m + 5{\cdot}99. \qquad (c)$$
$$y - x\tan i_m = -11{\cdot}70\tan i_m - 12{\cdot}50. \qquad (d)$$
$$y - x\tan i_m = -1{\cdot}54\tan i_m + 12{\cdot}20. \qquad (e)$$
$$y - x\tan i_m = -9{\cdot}37\tan i_m - 5{\cdot}09. \qquad (f)$$
$$y - x\tan i_m = -11{\cdot}80\tan i_m - 11{\cdot}10. \qquad (g)$$

On inspection of these seven equations, it will be seen that five of them form two natural groups, and that the remaining two must be treated separately:—

*1st Group*—(*a*) (*d*) (*g*).

$$y - x\tan i_m = -11{\cdot}5\tan i_m - 7{\cdot}11. \qquad (a)$$
$$y - x\tan i_m = -11{\cdot}7\tan i_m - 12{\cdot}50. \qquad (d)$$
$$y - x\tan i_m = -11{\cdot}8\tan i_m - 11{\cdot}10. \qquad (g)$$

*2nd Group*—(*b*) (*f*).

$$y - x\tan i_m = -8{\cdot}30\tan i_m - 6{\cdot}28. \qquad (b)$$
$$y - x\tan i_m = -9{\cdot}37\tan i_m - 5{\cdot}09. \qquad (f)$$

*Remaining equations*—(*c*) (*e*).

$$y - x\tan i_m = -3{\cdot}09\tan i_m - 5{\cdot}99. \qquad (c)$$
$$y - x\tan i_m = 1{\cdot}54\tan i_m - 12{\cdot}20. \qquad (e)$$

Taking the mean of the 1st group, we find

$$y - x \tan i_m = -11{\cdot}67 \tan i_m - 10{\cdot}24.$$

Taking the mean of the 2nd group, we find

$$y - x \tan i_m = -8{\cdot}83 \tan i_m - 5{\cdot}68.$$

From these equations, we find

$$\tan i_m = -1{\cdot}60,$$

$$y - x \tan i_m = +8{\cdot}43\ ;$$

and from the aberrant equations (*c*) and (*e*), we find

$$\tan i_m = -\ 1{\cdot}34,$$

$$y - x \tan i_m = +10{\cdot}13.$$

As the first results depend upon five equations and the second upon two equations, the mean result of all seven equations is

$$\tan i_m = -1{\cdot}526,$$

$$y - x \tan i_m = +8{\cdot}916 \text{ inches.}$$

Additional information may be had from equation (4), viz. :

$$(A_2 - x\cos^2\bar{\sigma})^2 + (B_2 - y\cos^2\bar{\sigma})^2 = M^2\left(\frac{p}{p_m}\right)^6 \cos^4\bar{\mu}.$$

Expanding this equation, we find

$$(x^2+y^2)\cos^4\bar{\sigma} - 2A_2\cos^2\bar{\sigma}\,x - 2B_2\cos^2\bar{\sigma}\,y + (A_2^2+B_2^2) = M^2\left(\frac{p}{p_m}\right)^6\cos^4\bar{\mu}.$$

There are, fortunately, two weeks of hourly observation, one near the

equinox and the other near the solstice, in which the right-hand side of the equation is practically the same on corresponding days of the week.

*Week near Equinox.*

1875.

Sept. 29. $(a)$ $0{\cdot}996\,(x^2+y^2) - 1{\cdot}996\,(11{\cdot}5\,x - 7{\cdot}1\,y) + 182{\cdot}66 = 0{\cdot}984\,M'^2$.

„ 30. $(b)$ $0{\cdot}994\,(x^2+y^2) - 1{\cdot}994\,(13{\cdot}8\,x - 5{\cdot}8\,y) + 224{\cdot}08 = 0{\cdot}933\,M'^2$.

Oct. 1. $(c)$ $0{\cdot}992\,(x^2+y^2) - 1{\cdot}992\,(13{\cdot}6\,x - 2{\cdot}7\,y) + 192{\cdot}25 = 0{\cdot}852\,M'^2$.

„ 2. $(d)$ $0{\cdot}990\,(x^2+y^2) - 1{\cdot}990\,(13{\cdot}3\,x + 1{\cdot}6\,y) + 179{\cdot}45 = 0{\cdot}767\,M'^2$.

„ 3. $(e)$ $0{\cdot}988\,(x^2+y^2) - 1{\cdot}988\,(10{\cdot}6\,x + 3{\cdot}3\,y) + 123{\cdot}25 = 0{\cdot}689\,M'^2$.

„ 4. $(f)$ $0{\cdot}986\,(x^2+y^2) - 1{\cdot}986\,(\ 9{\cdot}0\,x + 4{\cdot}8\,y) + 104{\cdot}04 = 0{\cdot}630\,M'^2$.

„ 5. $(g)$ $0{\cdot}984\,(x^2+y^2) - 1{\cdot}984\,(\ 6{\cdot}4\,x + 5{\cdot}6\,y) + \ 72{\cdot}32 = 0{\cdot}597\,M'^2$.

$M'^2$ is affected by the lunar parallax, which ranges in the week from 55·7 to 54·2.

*Week near Solstice.*

1875.

Dec. 20. $(a')$ $0{\cdot}707\,(x^2+y^2) - 1{\cdot}682\,(-7{\cdot}0\,x + 1{\cdot}8\,y) + \ 52{\cdot}24 = 0{\cdot}972\,M'^2$.

„ 21. $(b')$ $0{\cdot}707\,(x^2+y^2) - 1{\cdot}683\,(-6{\cdot}4\,x - 2{\cdot}3\,y) + \ 46{\cdot}25 = 0{\cdot}914\,M'^2$.

„ 22. $(c')$ $0{\cdot}708\,(x^2+y^2) - 1{\cdot}684\,(-4{\cdot}8\,x - 5{\cdot}5\,y) + \ 53{\cdot}29 = 0{\cdot}835\,M'^2$.

„ 23. $(d')$ $0{\cdot}709\,(x^2+y^2) - 1{\cdot}685\,(-2{\cdot}8\,x - 8{\cdot}5\,y) + \ 80{\cdot}09 = 0{\cdot}751\,M'^2$.

„ 24. $(e')$ $0{\cdot}710\,(x^2+y^2) - 1{\cdot}686\,(+1{\cdot}9\,x - 8{\cdot}3\,y) + \ 72{\cdot}50 = 0{\cdot}677\,M'^2$.

„ 25. $(f')$ $0{\cdot}711\,(x^2+y^2) - 1{\cdot}687\,(+4{\cdot}4\,x - 9{\cdot}5\,y) + 109{\cdot}61 = 0{\cdot}624\,M'^2$.

„ 26. $(g')$ $0{\cdot}712\,(x^2+y^2) - 1{\cdot}688\,(+6{\cdot}9\,x - 7{\cdot}3\,y) + 100{\cdot}90 = 0{\cdot}599\,M'^2$.

The moon's parallax ranged from 55·7 to 54·2 during the week. Hence

the right-hand side of the equations, in both weeks, are practically equal from day to day.

Equating these in pairs, we find (neglecting small differences),

$(a\,a')$

$$0{\cdot}289\,(x^2+y^2) - 34{\cdot}64\,x + 17{\cdot}15\,y + 130{\cdot}42 = 0.$$

$(b\,b')$

$$0{\cdot}287\,(x^2+y^2) - 38{\cdot}21\,x + 7{\cdot}68\,y + 177{\cdot}83 = 0.$$

$(c\,c')$

$$0{\cdot}284\,(x^2+y^2) - 35{\cdot}12\,x - 3{\cdot}87\,y + 138{\cdot}96 = 0.$$

$(d\,d')$

$$0{\cdot}281\,(x^2+y^2) - 31{\cdot}20\,x - 17{\cdot}55\,y + 99{\cdot}36 = 0.$$

$(e\,e')$

$$0{\cdot}278\,(x^2+y^2) - 17{\cdot}94\,x - 20{\cdot}59\,y + 50{\cdot}75 = 0.$$

$(f\,f')$

$$0{\cdot}275\,(x^2+y^2) - 10{\cdot}47\,x - 25{\cdot}60\,y + 31{\cdot}54 = 0.$$

$(g\,g')$

$$0{\cdot}272\,(x^2+y^2) - 1{\cdot}01\,x - 23{\cdot}43\,y - 28{\cdot}58 = 0.$$

As the coefficients of these equations depend on $\cos^2\mu$ and $\cos^4\mu$, which pass through all their changes in seven to eight days, we may add all together, and find

$$1{\cdot}966\,(x^2+y^2) - 168{\cdot}59\,x - 66{\cdot}08\,y + 600{\cdot}28 = 0.$$

or,

$$(x^2+y^2) - 85{\cdot}5\,x - 33{\cdot}5\,y + 300 = 0.$$

This is the final results from heights.

Combining this with

$$y = -1{\cdot}526\,x + 8{\cdot}916 \quad \text{derived from times, we find}$$

$$3{\cdot}33\,x^2 - 61{\cdot}46\,x + 86 = 0.$$

$$x^2 - 18{\cdot}4\,x + 25{\cdot}8 = 0.$$

$$x \left\langle \begin{matrix} +\,16{\cdot}87 \\ +\,\;1{\cdot}53. \end{matrix} \right.$$

Hence, finally

$$x = 1{\cdot}53,$$

$$y = 6{\cdot}57.$$

$$S = 6{\cdot}745 \text{ inches}; \qquad \tan 2i_s = 4{\cdot}294.$$

$$M = 14{\cdot}742 \text{ inches}; \qquad \tan 2i_m = 1{\cdot}526.$$

$$\frac{S}{M} = 0{\cdot}4575.$$